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Journal of Sound and Vibration 282 (2005) 151–162

JOURNAL OF SOUND AND VIBRATION

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Motion of droplets on solid surface using acoustic radiation pressure

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Received 19 May 2003; accepted 16 February 2004
Available online 17 September 2004

Abstract

This paper is concerned with a practical method to displace a droplet along a vibrating beam. This method is based on the movement of the droplet sliding toward an anti-node of vibration. These phenomena are explained by the action of acoustic radiation pressure. The displacement along the beam is obtained by using different modes of vibration. The vibration level required to move the droplet is compared with the one needed for its spraying. The physical significance of our results is discussed.

1. Introduction

Various papers present some works on the interaction of an ultrasonic beam with bubbles, with a suspended droplet and water–air interface [1,2]. There are also studies on the interaction of vibrations with powder [3]. Hashimoto et al. [4,5] presented a paper on acoustic levitation of planar specimen using flexural vibration.

This work deals with a different topic, the interaction of the acoustic field produced by flexural vibration with liquids. The interaction between solids and flexural wave is well known and its main application concerns the ultrasonic motors since 1982, but the principles used in this case cannot be extended to a solid–liquid interaction.

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2004.02.020

In a previous paper (see Ref. [6]), the effect of flexural plate waves on liquid has already been studied. The first experiment was about the effect of an acoustic wave on water. A thin layer of water is placed on a metallic beam and at rest, its surface is flat. When a standing flexural vibration is produced in the structure, a permanent deformation was obtained if the vibration level was high enough. A model had also been developed taking into account the hydrostatic pressure, the capillary pressure induced by the surface tension σ and the acoustic radiation pressure.

The model shows that the acoustic radiation pressure can explain the permanent deformation of the surface. This acoustic radiation pressure, which is a second order phenomenon, varies according to the square of the vibration amplitude. It is rather weak but sufficient to explain the observed behaviour. The acoustic streaming is not taken into account because its effect is negligible at the working frequency (about 30 kHz).

A second work about water droplet displacement has been published [7]. In this case, a standing wave is generated in the structure. When the amplitude of the vibration is high enough, a droplet not located at an anti-node moves towards the nearest one. The model has been improved to explain the displacement of the droplet towards the anti-node of a standing wave. To sum up, the model shows that it is possible to move droplets with dimensions ruled by the wavelength. The best condition is obtained when the drop contact diameter is about a quarter of the flexural wavelength. However, the contact angle should not be too small to prevent very large adhesion of the droplet onto the substrate.

A possible application of this phenomenon is the continuous displacement of a water droplet using a moving standing wave. This displacement could by used for a bio-chemical analysis system. The experiment has been achieved using a "caterpillar"-like structure made up of a beam looped to itself, in order to produce a continued structure. This type of component allows the production of a standing wave at any position, since the structure has no end. The vibration is excited with four piezoelectric ceramics. Two ceramics on the left side are used to excite a given standing wave. The other pair of ceramics can be used to excite another standing wave, shifted by a quarter of a wavelength. This shift corresponds to the maximum drop displacement step that can be achieved. This is because the drop can move towards the anti-node of the acoustic displacement, as will be described later. Some success has been obtained with the "caterpillar" device but this kind of device is not easy to use for various reasons. Gluing flat ceramics was unsuitable to withstand a sufficient level of vibration because the glue was rapidly destroyed. Moreover, with bolted bulk ceramics, it is very difficult to change the standing wave position because the ceramics are clamped and produce boundary conditions. A new device that uses different standing waves with different frequencies has been designed. Furthermore, some experiments will be presented to compare the levels of vibration required to move and spray the droplet.

2. Experiment

The new device is a beam with two stacks of multi-layers piezoelectric actuators on each end (Fig. 1). In this device, several high order vibration modes are excited one after the other to change the position of the anti-nodes and so, to move the droplet along the beam. Since high

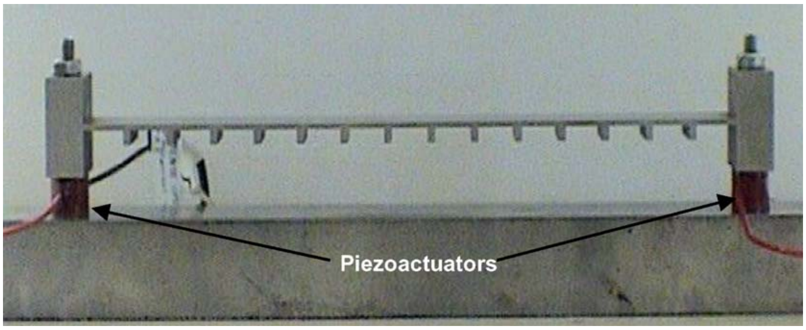

Fig. 1. Photography of the periodic device with piezoelectric stacks.

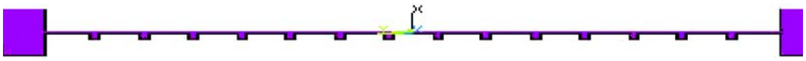

Fig. 2. Periodic vibrating beam (side sight).

order flexural modes are used to obtain the right wavelength, other types of modes appear and disturb the ones needed. A periodic structure to eliminate disturbing modes has been tested (Fig. 2).

2.1. Disturbing mode elimination

The vibration modes are sorted according to the number of anti-nodes in the direction of length and in the direction of width. Let us note (n,m) the mode with n anti-nodes along the length and m anti-nodes in the direction of width.

To perform a good displacement of a drop along the length of the beam, the flexural modes (n,1) have to be used. In a usual clamped–clamped beam the frequencies of modes (n,1) and (n,2) are presented in Fig. 3. It can be seen that, for instance, the frequencies of modes (9,1) and (7,2) are very close. Consequently, mode (7,2) disturbs the mode of pure flexural vibration.

In order to improve the performance, the behaviour of a beam with a periodic structure (Figs. 1 and 2) is simulated using finite element computations. The parameters depicted in Fig. 4 are α, the ratio of length l of a cell by the period L_0 of the vibrating structure and k, the ratio of the thickness of the cell by the thickness of the membrane. The total length $L = NL_0$. The results showed the elimination of the disturbing modes with such a structure. Finally the following data have been chosen: $N = 14$, $L_0 = 11.3\,\text{mm}$, $e = 1.1\,\text{mm}$, $b = 20\,\text{mm}$, $k = 2$, $\alpha = \frac{2}{11}$. The material used is stainless steel with a density $\rho = 7900\,\text{kg}\,\text{m}^{-3}$, Young's modulus $E = 2.1 \times 10^{11}\,\text{Pa}$ and Poisson's ratio $\upsilon = 0.3$. These dimensions and these parameters allow to obtain half the wavelength close to a centimetre and frequencies ranging between 20 and 40 kHz with enough amplitude of vibration to displace a drop. This justifies the number of cells ($L/14 \approx 12\,\text{mm}$, $L/18 \approx 9\,\text{mm}$) and the low values of k and α. Indeed, for a length of 170 mm and a thickness of 1.1 mm the mode with half a wavelength of about one centimetre is close to rank 14.

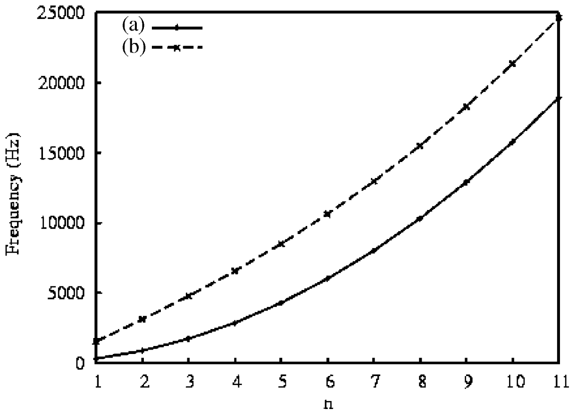

Fig. 3. Eigenfrequencies of vibrating modes (n,1) and (n,2) as a function of rank n for a usual beam: (a) modes (n,1); (b) modes (n,2).

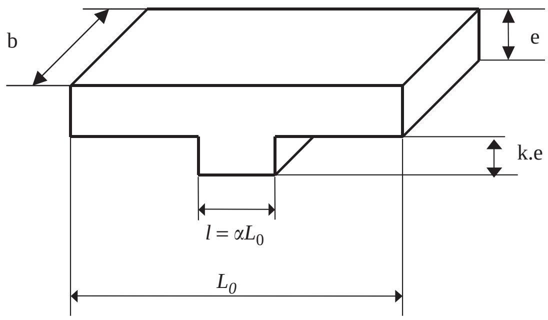

Fig. 4. Description of the dimensions of a cell of the periodic structure.

With this configuration, the results in Fig. 5 show a band gap in the frequencies of the modes (n,2) for n between 14 and 15. Then, the disturbing modes (n,2) are sufficiently eliminated to test the drop displacement by switching the modes (n,1) in this gap of frequencies.

2.2. Experimental results about liquid drop displacements

The modes given in Table 1 are used in order to move the drop along the beam. An arbitrary number between 1 and 6 is used to indicate the mode. In the experiment, the drop is always placed at the same initial position on the beam, which corresponds to an anti-node of the mode. Then, the nearest equilibrium position of the drop to move to the desired direction has to be selected. If two modes can displace the droplet in the same direction, the one that offers the largest displacement is chosen. Hence, a succession of modes allows to make the drop move step by step. Fig. 6 shows the equilibrium position of each anti-node for each flexural mode. The solid lines correspond to the modes used for the displacement of the droplet; the dashed lines correspond to existing anti-nodes but not used.

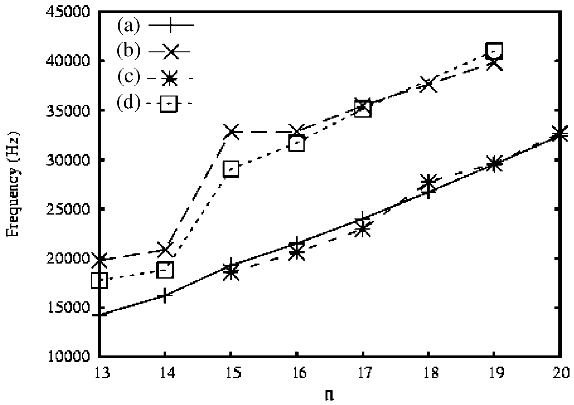

Fig. 5. Experimental and theoretical eigenfrequencies of the periodic beam modes: (a) modes (n,1) simulated; (b) modes (n,2) simulated; (c) modes (n,1) measured; (d) modes (n,2) measured.

Table 1
Frequencies for different modes

Mode rank	Frequency (Hz)	Mode number
16	22,986	1
17	25,365	2
18	27,751	3
19	30,223	4
20	32,705	5
21	35,112	6

The mode number is an arbitrary number used to select this mode.

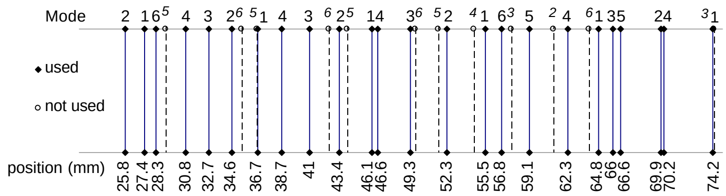

Fig. 6. Position of the anti-nodes of the modes used for the experimental displacement of the droplet. ♦, modes used for displacement; ○, modes unused.

This sequence had been used to move a 30 μl water droplet with an apparent diameter on a beam of 5 mm. When the droplet is in the last position of the sequence, it can be moved back to the initial position by switching the frequencies in the reverse way. To achieve the displacement of the droplet with the lowest power drive, the frequency of the excited mode must be adjusted. Indeed, the frequency of a mode can change with the droplet position. The contribution of the droplet mass is greater as the vibration amplitude of the beam is high at the droplet localization. This load on the beam, represented by the liquid mass, reduces the resonant frequency of the flexural mode. This effect is emphasized when the droplet is placed close to an anti-node compared to the contribution of the droplet mass when the drop is placed near a node. Therefore, the frequency decreases when the drop moves toward the anti-node. This change is not large but is greater than the halfwidth of the resonance peak of the mode. Consequently, the frequency must be adjusted to obtain a more efficient displacement of the drop. This adjustment can be performed automatically without difficulty if the sweep frequency range is smaller than the difference with the frequency of the next position. The sweep direction (decreasing or increasing the frequency) does not produce the same effect. The frequency of the mode is a little lower when the drop is at the final position (about 10 Hz on the anti-node). So it is more efficient to decrease the frequency because when the drop begins to move it will produce a reduction of the frequency.

Secondly, the length of the beam affects the range of the drop size. Indeed, the droplets can move if the diameter is about the quarter of the wavelength of the vibration (see Ref. [7]). In order to move smaller droplets smaller wavelengths are needed. It is also difficult to move the drop at the end or at the middle of the beam because the anti-nodes of the modes are too close to each other.

2.3. Amplitude for drop displacement and drop spraying

The amplitude of the beam vibration was measured using a heterodyne interferometer. There are no obvious relations between the amplitude of beam vibration and the amplitude of acoustic wave inside the liquid. Furthermore, there are no more obvious relations between the amplitude of beam vibration and the amplitude of the capillarity surface wave that can appear on the surface of the droplet. This capillary wave is often considered as the origin of the drop atomization, as it will be further discussed. It is assumed that the possible proportionality between the different wave amplitudes allows us to take into account the amplitude of the beam vibration as a reference for the discussion.

The amplitude required to move a drop and the amplitude required to spray the same drop have been measured. Five different modes with a frequency between 23 and 33 kHz are excited in the beam. The liquid used here is water and various water–glycerol mixtures (20% and 50% of glycerol). Fig. 7 shows the amplitude of the beam vibration A_d required for the displacement and A_n for setting up the spraying of a droplet.

The theoretical curves are related to the amplitude of acoustic vibration and not directly to the amplitude of beam vibration. Therefore, the theoretical curves A_n^* (water) and A_d^* (water) are fitted on experimental data by adjustment of a constant and give only the behaviour as a function of the frequency. In some cases, A_d is not so far from A_n and it also shows that the displacement condition is not modified by the change of viscosity and the amplitude required decreases with the frequency. This result agrees with the model published in [7]. It is shown here that the viscosity is not a fundamental parameter and that the amplitude of vibration required varies as $f^{-3/4}$ which is not far from the present experiment.

For the spray creation, the amplitude A_n is given as a function of frequency f by

$$A_n = \sqrt{\frac{\alpha}{f^{4/3}} + \frac{\beta}{f}} \tag{1}$$

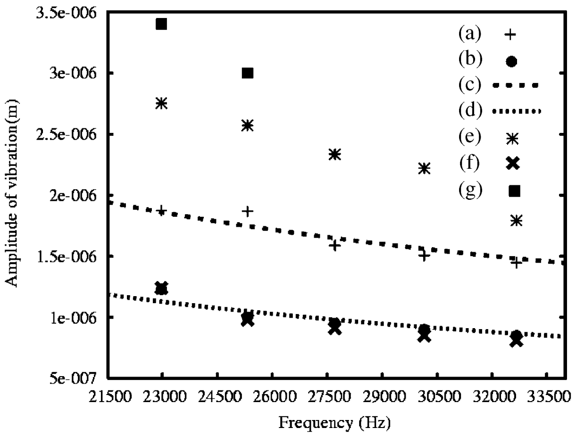

Fig. 7. Amplitude of vibration A_d required for the displacement of a droplet and amplitude A_n required for the spraying of a droplet for different liquids: (a) A_n (water); (b) A_d (water); (c) A_n^* (water-theory); (d) A_d^* (water-theory); (e) A_n (20% glycerol); (f) A_d (50% glycerol); (g) A_n (50% glycerol). The liquids used are 1—water, 2—water 80%, glycerol 20% and 3—water 50%, glycerol 50%.

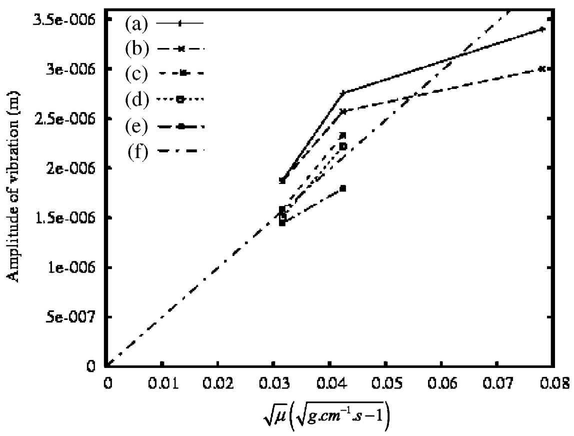

Fig. 8. Amplitude required for droplet spraying as a function of the square root of the viscosity: (a) $f = 23,000$ Hz; (b) $f = 25,350$ Hz; (c) $f = 27,000$ Hz; (d) $f = 30,150$ Hz; (e) $f = 32,700$ Hz; (f) model.

with α proportional to $(\sigma/\rho)^{2/3}$ and β proportional to μ/ρ where σ is the superficial tension, ρ the density and μ the viscosity.

In the following paragraph the origin of Eq. (1) will be explained and the value of the coefficients will be given but right now, we can say that the frequency dependence found experimentally is close to the model.

When the experiment is carried out with water–glycerol mixture it seems that the variation of the amplitude is mainly related to the change of viscosity which requires a ratio β/α greater than the value given in our model. Nevertheless, the experimental results show the proportionality with the square root of the viscosity as in the model (see Fig. 8). Let us remark that the variation of viscosity can also modify the relation between beam vibration amplitude and the capillarity wave amplitude. As a preliminary conclusion, the difference between the displacement threshold and the spraying threshold is rather small for low-viscosity liquids (water) but the difference is sufficient to use the device to move a drop.

3. Discussion

Several points can be discussed after this experimentation like the level of acoustic pressure radiation required to move a drop, the comparison between the influence of the acoustic pressure radiation and the acoustic streaming and finally a model for the spray of liquid. First, let us show that the acoustic radiation pressure required is not far from the maximum radiation possible in water. Let u_0 be the vibration amplitude and v the maximum vibration speed of the particle ($v = u_0\omega$).

The acoustic pressure is given by

$$P = \rho k u_0 c^2 = \rho c u_0 \omega. \tag{2}$$

The acoustic radiation pressure is given in first approximation by $P_r = Pv/c$ where c is the sound velocity. This expression is an approximation of the expression given in Ref. [8].

Then

$$P_r = \rho k \frac{v}{c} u_0 c^2 = \rho \omega^2 u_0^2. \tag{3}$$

By elimination of u_0 between Eqs. (2) and (3) and with $c = \omega/k$

$$P_r = \frac{P^2}{\rho c^2} \tag{4}$$

with $P = 10^5\,\mathrm{Pa}$, $\rho = 1000\,\mathrm{kg\,m^{-3}}$ and $c = 1500\,\mathrm{m\,s^{-1}}$, then $P_r = 5\,\mathrm{Pa}$.

The previous result is very rough but it shows that the range of the acoustic radiation pressure required for the observed effects is not far from the maximum possible.

Let us remark that in the air the result with $P = 10^5\,\mathrm{Pa}$, $\rho = 1\,\mathrm{kg\,m^{-3}}$ and $c = 300\,\mathrm{m\,s^{-1}}$, will be $P_r = 1.1 \times 10^5\,\mathrm{Pa}$.

Secondly, comparing the radiation acoustic pressure with the acoustic streaming in water for a frequency of about 30 kHz and according to Rife et al. [9], the force per volume unit due to acoustic streaming is given by

$$F = \frac{I \mathrm{e}^{-x/l_\mu}}{c l_\mu} \approx \frac{I}{c l_\mu} \tag{5}$$

for $x \ll \mu$, x is the abscissa along propagation direction and l_μ is the characteristic absorption length. The acoustic intensity is

$$I = \frac{1}{2}\frac{P^2}{\rho c} = \tfrac{1}{2} P_r c \tag{6}$$

with P_r being the acoustic radiation pressure and c the sound velocity. Then during the first part of sound propagation, the equivalent pressure due to acoustic streaming $P_{\mathrm{eq}} = Fx$ is obtained as a function of P_r where x is the length of the sound propagation:

$$P_{eq} = \frac{x}{2l_\mu} P_r. \tag{7}$$

Evaluating the ratio x/l_μ allows to compare acoustic radiation pressure and acoustic streaming. According to Ref. [10] the absorption coefficient A in water at a frequency $f = 1\,\text{MHz}$ is $A = 0.025\,\text{Neper/m}$. Then for $f = 1\,\text{MHz}$, $l_\mu = 40\,\text{m}$. Additionally, the absorption increases as f^2 then for $f = 30\,\text{kHz}$ the absorption is negligible. Obviously the result would be different for $f = 100\,\text{MHz}$ when $l_\mu = 4\,\text{mm}$ which is 10^4 times smaller.

One remaining problem is the spray formation. Let us discuss this problem. First, the previous discussion shows that the required acoustic levels are close to the level where the cavitations occurs. Anyway, even if the cavitations do not appear, the acoustic level is not far from the level required to reach an unstable level for the capillary wave. If it is supposed that in order to obtain droplet atomization the vibration energy in the droplet volume must be equal to the energy required to form this droplet, then the energy is the sum of two terms, one due to capillarity and the other due to viscosity. If V_g is the droplet volume, S_g the surface of the droplet, v the amplitude of the particle speed, E_v is the vibration energy and E_{st} the energy corresponding to superficial tension, then

$$E_v = \tfrac{1}{2}\rho v^2 V_g, \tag{8}$$

$$E_{st} = \sigma S_g. \tag{9}$$

Now let us compute an evaluation of the lost energy due to the viscosity in the drop formation. It is assumed that the speed of the liquid in the drop during its formation $v(r)$ is like the speed distribution in a laminar flow that is to say given by [11]

$$v(r) = \frac{-v_0}{r_0^2} r^2 + v_0 \tag{10}$$

in such a manner that $v(r = 0) = v_0$, and $v(r = -r_0) = 0$ (r_0 is the radius of the drop) (see Fig. 9).

The pressure in the drop for a length l is given by

$$\Delta P = \mu \frac{\partial^2 v}{\partial r^2} l \tag{11}$$

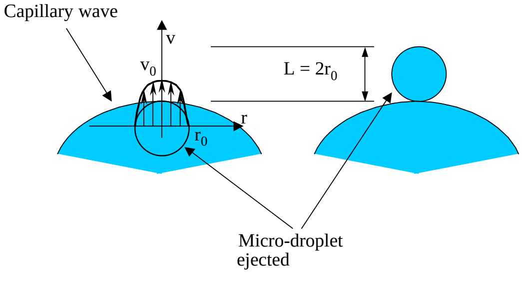

Fig. 9. Schematic of a microdroplet ejected during spraying.

with l about $2r_0$, then

$$\Delta P = 4\mu_0 \frac{v_0}{r_0} \tag{12}$$

or

$$F = \Delta P \pi r_0^2 = 4\pi\mu_0 v_0 r_0 \tag{13}$$

then the energy dissipated for the micro-droplet formation at the surface of the liquid is about

$$E_{\text{vis}} = 8\pi\mu_0 v_0 r_0^2. \tag{14}$$

Furthermore $v_0 = r_0/\tau$ then

$$E_{\text{vis}} = 16\pi\mu_0 \frac{r_0^3}{\tau}, \tag{15}$$

where τ is the characteristic time for the micro-droplet formation that is to say $\tau = T/2 = 1/(2f)$ then

$$E_{\text{vis}} = 32\pi\mu_0 r_0^3 f. \tag{16}$$

As a result, the equality between vibration energy and energy required to form the micro-droplet is expressed as

$$\tfrac{1}{2}\rho(A\omega)^2\left(\tfrac{4}{3}\pi r_0^3\right) = 4\pi\sigma r_0^2 + 32\pi\mu_0 r_0^3 f. \tag{17}$$

Using the relation between micro-droplet radius and the wavelength of the capillary wave [12] combined with the dispersion relation of the wave, that is to say

$$2r_0 = 0.34\lambda \approx \frac{\lambda}{3}, \tag{18}$$

$$\frac{\omega}{k} = \sqrt{\frac{\sigma.k}{\rho}}, \tag{19}$$

then

$$A = \sqrt{\left(\frac{18}{(2\pi)^{7/3}}\left(2\left(\frac{\sigma}{\rho f^2}\right)^{2/3} + \frac{8\mu_0(2\pi)^{1/3}}{3\rho f}\right)\right)}. \tag{20}$$

With the water at $f = 30\,\text{kHz}$ with $\sigma = 7.2\times10^{-2}\,\text{J m}^{-2}$, $\rho = 1000\,\text{kg m}^{-3}$ and $\mu_0 = 10^{-3}$ Pa.s, then $A = 30\,\mu\text{m}$. With such frequency, $\lambda = 80\,\mu\text{m}$ therefore $A = 0.38\,\lambda$. Let us remark that the sub-harmonic excitation of the surface wave has not been introduced (see Ref. [13]) because it is not obvious that in the present case the surface wave is excited by a parametric phenomenon. The excitation can also be due to the transverse component of the plate vibration on the edge of the drop. In any case, the difference does not change the conclusion fundamentally.

The same relation as the previous one (Eq. (17)) can be rewritten when the viscosity is neglected, as

$$d = 2r_0 = \frac{3\sigma}{\rho(\pi A f)^2}. \tag{21}$$

This is exactly the relation given by Drobe and Bolle [14].

In order to evaluate the difficulty in obtaining such a wave, the amplitude of an acoustic wave in water with the same energy flux is computed. The relation is

$$A_s^2 \omega^2 c = A_c^2 \omega^2 v_g \tag{22}$$

with A_s and c, respectively, being sound wave amplitude and its velocity, A_c and v_c, the capillary wave amplitude and phase velocity of capillary wave. In water $c = 1500\,\mathrm{m\,s^{-1}}$ and for a frequency $f = 30\,\mathrm{kHz}$, we obtain $v_c = 2.37\,\mathrm{m\,s^{-1}}$, the group velocity $v_g = (3/2)v_c = 3.55\,\mathrm{m\,s^{-1}}$ and $A_s = A_c/20.5$ then $A_s = 1.5\,\mu\mathrm{m}$ for $A_c = 30\,\mu\mathrm{m}$.

This value A_s is in the order of the amplitude required to obtain an acoustic displacement of the droplet.

Comparing the energy corresponding to tension energy $E_{\mathrm{ten}} = 4\pi\sigma r_0^2$ and the energy, corresponding to viscosity $E_{\mathrm{vis}} = 32\pi\mu_0 r_0^3 f$, with $f = 30\,\mathrm{kHz}$, $\sigma = 72 \times 10^{-3}\,\mathrm{J\,m^{-2}}$, $\mu_0 = 10^{-3}\,\mathrm{Pa.s}$, $\lambda = 80\,\mu\mathrm{m}$, $r = 0.15 \times 80 = 12\,\mu\mathrm{m}$, we get $E_{\mathrm{vis}} = 5.21 \times 10^{-12}\,\mathrm{J}$, $E_{\mathrm{ten}} = \sigma 4\pi r^2 = 1.31 \times 10^{-10}\,\mathrm{J}$ and then $E_{\mathrm{vis}}/E_{\mathrm{ten}} = 0.04$.

In water for our frequency, the viscosity seems to be negligible, but if the viscosity of another liquid is only 25 times greater than in water, it is no longer true.

From this previous evaluation, it can be noted that the vibration energy required to move the droplet is not far from the energy required to achieve spray formation due to cavitations or capillary wave instability. This evaluation is confirmed experimentally.

Let us also remark that our evaluation cannot be used to settle on if the spray formation is due to either cavitation or capillary wave instability. As a matter of fact, relating frequency and droplet radius is not very conclusive because even if the primary phenomenon is not the capillary wave itself then there is a relation between droplet radius and capillarity. The relation between wavelength of capillarity wave and droplet radius does not necessarily indicate a causal relation.

4. Conclusion

The droplet can be efficiently displaced with a vibrating beam. The vibration level is rather high and not far from the level required for spraying the droplet. Nevertheless, the difference is sufficient to prevent the spraying if necessary. An application of this method is the handling of droplet for bio-chemical analysis.

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